IS IT POSSIBLE TO ANSWER THE QUESTION "IS THE UNIVERSE OPEN OR CLOSED?"

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ABSTRACT

It is shown that all cosmological data collected to date are insufficient to distinguish between an open and a closed universe. Further, unless the results of certain experiments are exceedingly fortuitous, no experiment conducted or proposed to date <u>can</u> distinguish between these two possibilities.

Evidence showing that the present day deceleration parameter q_0 is definitely less than ½ has been accumulating rapidly, (Wagoner 1973; Gott, Gunn, Schram, and Tinsley 1974; Pasachoff and Fowler 1974) and the condition $q_0 < \frac{1}{2}$ is generally thought to imply an unbounded universe - a universe which will expand forever. I shall show, on the contrary, that this condition is insufficient to prove the universe will expand forever, since there exists an infinitesimal perturbation of the form of the matter tensor which can change any expanding universe into a contracting one.

To see the physical significance of this, consider the procedure used to determine which matter tensor is to be used in the field equations of general relativity. We make observations of the behavior of matter in a local Lorentz frame on earth where tidal forces are very small (so that curvature coupling can be neglected) and from these observations we construct for <u>all</u> forms of ponderable matter and <u>all</u> fields a Lorentz covariant matter tensor $T_i^{\hat{\mu}\hat{\nu}}$ valid in this frame. (carets over the indicies will denote a local Lorentz frame) For each type of matter we now find a tensor $T_i^{\mu\nu}$ which is generally covariant but which has one of the $T_i^{\hat{\mu}\hat{\nu}}$ as components in a local Lorentz frame; we assume that these $T_i^{\mu\nu}$ are the only possible types of matter tensors at <u>every</u> point in spacetime. These different types of matter will interact with gravitation via the Einstein equations $G^{\mu\nu} = 8\pi T^{\mu\nu}$, where $T^{\mu\nu}$ is some combination of the individual $T_i^{\mu\nu}$'s. To calculate the future history of the universe from these equations, we will have to evaluate $T^{\mu\nu}$ by determining the <u>amount</u> of each different field or type of ponderable matter in the universe at the present time from measurements in $J^-(E)$, the causal past of the earth (Misner, Thorne, and Wheeler 1973).

Note that two properties of $T^{\mu\nu}$ must be determined by experiment: first, what types of matter are present in the universe; i.e., what is the form of $T^{\mu\nu}$? Secondly, how much of each type of matter is actually present - which types of matter are dominating the large scale evolution of the universe? The important point is simply this: since $T^{\mu\nu}$ is determined by experiment, it cannot be known exactly. Therefore, any prediction of the future behavior of the universe which depends on knowing the exact matter tensor is unreliable; a physically realistic spacetime must be stable under infinitesimal perturbations of the form (or content) of the matter tensor.

Thus, if there exists a perturbation $(M^{\mu\nu} + T^{\mu\nu})$ of the matter tensor, with $|M^{\hat{\mu}\hat{\nu}}|$ greater than zero but still arbitrarily small in any local Lorentz frame at every point in spacetime, which has the effect of closing a universe with $q_0 < \frac{1}{2}$, then we must declare any prediction of openness based on $q_0 < \frac{1}{2}$ unreliable; the model of spacetime that yields this prediction is physically unrealistic.

And there is an arbitrarily small perturbation which has this effect. In a local Lorentz frame it can be written.

$$M^{\hat{\mu}\hat{\nu}} = \epsilon\, \eta^{\hat{\mu}\hat{\nu}} \qquad , \quad \epsilon > 0 \tag{1}$$

where $\eta^{\hat{\mu}\hat{\nu}}$ is the Minkowski tensor and ϵ is a constant with as small a magnitude as we please. (Note that although the local energy density, $M^{\hat{o}\hat{o}} = -\epsilon$, is negative, the field defining (1) will not give rise to quantum mechanical instabilities since the local energy density is the same at all points of spacetime. It cannot change.) By choosing ϵ sufficiently tiny, we can make the changes induced in $J^-(E)$ by the perturbation completely undetectable by observers limited to information contained in $J^-(E)$ - the changes will be completely undetectable by us. (Experiments by Partridge (1973) based on the absorber theory of radiation are in effect measurements on the future, but the results of his work are equivocal with respect to the closure of the universe.) (Gott, Gunn, Schram, and Tinsley 1974; Professor X 1967) However, this small change will have an enormous effect on the future evolution of the universe; it will eventually turn an expanding universe into a contracting one. To show this, it will be necessary to give precise definitions of the concepts "expanding" and "contracting", definitions which are as close as possible to the ordinary meanings of these words. I shall say "the universe is expanding at time t_0" if the expansion θ (see Hawking and Ellis 1973) for the definition of θ) of the timelike geodesics normal to the spacelike hypersurfaces defined by setting the universal time parameter t equal to t_0 satisfies $\theta > 0$; if $\theta < 0$, the universe will be said to be "contracting at time t_0 " (The existence of a universal time parameter is guaranteed by stable causality. (Hawking and Ellis 1973)) These are consistent with the usual definitions of expansion and contraction, since galactic clusters are used to measure the universe's expansion, and the clusters are generally assumed to move along geodesics.* (I will assume that the matter field (1) can interact with other forms of matter only via gravitation.)

The expansion θ of the geodesics obeys the equation (Hawking and Ellis 1973)

$$\frac{d\theta}{ds} = -R_{\mu\nu}V^\mu V^\nu - 2\sigma^2 - \frac{1}{3}\theta^2 + 2\omega^2$$
$$= -V^\mu V^\nu 8\pi\left(M_{\mu\nu} + T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}[M+T]\right)$$
$$-2\sigma^2 - \frac{1}{3}\theta^2 \qquad (2)$$

where V^{μ} is the tangent vector to a geodesic, σ is the shear of the geodesic congruence, s is the proper time measured along the geodesic (so as t increases, s increases), and the vorticity ω vanishes due to hypersurface orthogonality. Equation (2) implies

$$\frac{d\theta}{ds} \leq -8\pi\epsilon \tag{3}$$

provided we assume that the strong energy condition (Hawking and Ellis 1973) applies to $T^{\mu\nu}$. All known forms of matter satisfy this condition, but it is conceivable that we will detect a matter field which does not, say a matter field of the form

$$M'^{\mu\nu} = -\frac{\Lambda}{8\pi} g_{\mu\nu} \qquad , \Lambda > 0 \tag{4}$$

In this case, my argument will not go through, for $M'^{\mu\nu}$ would more than cancel $M^{\mu\nu}$ if ϵ were made arbitrarily small. However, there is absolutely no experimental evidence (Gott, Gunn, Schram, and Tinsley 1974) for $M'^{\mu\nu}$. Theoretical arguments for the existence of such matter (Zel'dovich 1968a and 1968b) are in general unable to determine the all important sign of Λ, though recent work by Dreitlein (1974) based on spontaneous symmetry breaking suggests $\Lambda < 0$.

Integrating (3), we obtain

$$\theta(s) - \theta(s_0) \leq -8\pi\epsilon(s - s_0) \tag{5}$$

with the geodesic proper time s set equal to s_0 on the initial hypersurface where $\theta(s_0) > 0$. Since the right hand side of (5) becomes arbitrarily negative as $s \to \infty$, so must $\theta(s)$, (assuming geodesic completeness, which should hold unless the geodesic runs into high density matter; i.e., unless the universe eventually contracts into a small volume) and this means the universe must eventually begin to contract. Clearly, this will occur no matter how small we take ϵ to be.

It is generally assumed that a universe which expands forever has noncompact spacelike hypersurfaces - it's unbounded in space- while a universe which will eventually contract is thought to have compact spacelike hypersurfaces. This is not necessarily true, for if the perturbation (1) is turned on, a universe will eventually contract whether its spacelike hypersurfaces are compact or not. Only if the universe were extremely small in the present epoch could we distinguish between these two possibilities, and the evidence strongly suggests that it is not extremely small. (Gott, Gunn, Schram, and Tinsley 1974)

Thus, I have shown that experiments conducted to date in $J^{-}(E)$ do not provide sufficient information to prove the universe open or closed. And unless we luckily detect the presence of a matter field of the form (4), the type of experiments conducted or proposed to date will never be able to settle the question of the closure of the universe.

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* Actually, the situation is a bit more complicated than this, for the matter flow lines may not be orthogonal to the chosen initial spacelike hypersurface. However, Hawking and Ellis (1965) have shown that if (a) the initial spacelike hypersurface is homogeneous, (b) the spacetime is globally hyperbolic, and (c) singularities do not occur except when matter densities are infinite, then the intersection of the geodesics normal to the hypersurface implies the intersection of the matter flow lines. The intersection of the matter flow lines in turn implies the contraction of the universe. If the universe as a whole is inhomogeneous, the concepts "expanding" and "contracting" no longer have any precise global meaning.

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Biographical Sketch

I was born and raised in Andalusia, a small town in southern Alabama. My interest in physics dates back to my kindergarten days (circa 1952) when I became fascinated with von Braun's visions of interplanetary flight. By the time I entered M.I.T. as an 18 year old freshman in 1965, however, this interest had metamorphosed into interest in fundamental physics, with particular attention to the role of Time in scientific theories. Graduating from M.I.T. in 1969, I became a graduate student at the University of Maryland, where I am now working toward a Ph.D. in General Relativity with Dieter Brill as thesis supervisor.

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